

**2103000206020031**  
**EXAMINATION FEBRUARY-MARCH 2024**  
**BACHELOR OF SCIENCE (SIXTH SEMESTER)**  
**MATHEMATICS-VI (MTH-601-RING THEORY)-LEVEL 2**

[Time: As Per Schedule]

[Max. Marks: 50]

**Instructions:**

**1. Fill up strictly the following details on your answer book**

- a. Name of the Examination : **BACHELOR OF SCIENCE (SIXTH SEMESTER)**
- b. Name of the Subject : **MATHEMATICS-VI (MTH-601-RING THEORY)-LEVEL 2**
- c. Subject Code No : **2103000206020031**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.
6. Use of non-programmable scientific calculator is allowed.

Seat No:

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Student's Signature

**Q.1 Answer the following as directed: (Any FIVE)**

**10**

- (1) Find the Unit element the commutative ring  $\langle R = \{\bar{0}, \bar{4}, \bar{8}, \bar{12}, \bar{16}\}, +_{20}, \times_{20} \rangle$ .
- (2) Is the commutative ring  $\langle R = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \bar{12}\}, +_{15}, \times_{15} \rangle$ . integral domain? Justify your answer.
- (3) Let  $\phi: R \rightarrow R'$  be a homomorphism of a ring  $R$  into a ring  $R'$ . Then prove that  $\phi(-a) = -\phi(a)$ ; for every  $a$  in  $R$ .
- (4) If  $U$  is an ideal of a ring  $R$  with a unit element  $1$  and  $1 \in U$ , then prove that  $U = R$ .
- (5) Show that  $3$  is not a greatest common divisor of  $6$  and  $12$  in the commutative ring  $3\mathbb{Z}$ .
- (6) Let  $R$  be a Euclidean ring and  $a \neq 0, b \neq 0$  in  $R$ . If  $b$  is unit in  $R$ , then prove that  $d(a) = d(ab)$ .
- (7) Show that  $\bar{4} \mid \bar{7}$  and  $\bar{7} \mid \bar{4}$  in the commutative ring  $J_9$ ; of integers modulo  $9$ .
- (8) If  $a$  in a Euclidean ring  $R$  is unit in  $R$ , then prove that  $d(a) = d(1)$ .

**Q.2 Answer any Two of the following: 10**

- (a) Define Field. Prove that every field is an integral domain.
- (b) If  $p$  is a prime number, then prove that  $J_p$ ; the ring of integers modulo  $p$ ; is a field.
- (c) In a Boolean ring  $R$ ; prove that: (i)  $a + a = 0$ ; for every  $a$  in  $R$ ;  
(ii)  $a \cdot b = b \cdot a$ ; for all  $a, b$  in  $R$ .

**Q.3 Answer any Two of the following: 10**

- (a) Let  $\phi: R \rightarrow R'$  be a homomorphism of a ring  $R$  into a ring  $R'$ . Then prove that  $\phi$  is an isomorphism if and only if  $I(\phi) = (0)$ ; where  $I(\phi)$  is the Kernel of a homomorphism  $\phi$ .
- (b) Define Ideal of Ring. If  $F$  is a field, then prove that its only ideals are and  $F$  itself.
- (c) Let  $\phi: R \rightarrow R'$  be a homomorphism of a ring  $R$  onto a ring  $R'$  with the Kernel  $U$ . Then prove that a mapping  $\Psi: R' \rightarrow R/U$  defined by  $\Psi(\phi(a)) = a + U$ ; for every  $\phi(a) \in R'$ ; is a one - one and onto homomorphism.

**Q.4 Answer any Two of the following: 10**

- (a) Define and illustrate Euclidean ring. Give the reason why the commutative ring  $J_{2024}$ ; of integers modulo 2024; is not a Euclidean ring.
- (b) Define Unit in a Commutative Ring with a Unit Element. If; for  $a, b$  in an integral domain  $R$  with a unit element; both  $a|b$  and  $b|a$  hold true, then determine the relation between  $a$  and  $b$  in  $R$ .
- (c) Define Greatest Common Divisor in Commutative Ring. Prove that two greatest common divisors of  $a, b$  in a Euclidean ring  $R$  are associates.

**Q.5 Answer any Two of the following: 10**

- (a) Define Prime Element and Relatively Prime Elements in a Euclidean Ring. Mention all the prime elements in the Euclidean rings  $\mathbb{Z}$  of all integers and  $J_{29}$ ; of integers modulo 29.
- (b) Prove, in a Euclidean ring  $R$ , that:
  - (i) If  $a, b$  and  $c$  are elements in  $R$  such that  $(a, b) = 1$  and  $a|bc$ , then  $a|c$ ;
  - (ii) If  $\pi$  is a prime element in  $R$ , then for any element  $a$  either  $(\pi, a) = 1$  or  $\pi|a$ .

- (c) State the second requirement in the definition of Euclidean ring. Let  $R$  be a Euclidean ring and  $a \neq 0, b \neq 0$  in  $R$ . If  $b$  is not unit in  $R$ , then prove that  $d(a) < d(ab)$ .

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